Comment on "On Porous-Wall Couette Flow under Slip Flow Conditions"

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IN a recent note, Inman[1] presents a solution for the laminar, incompressible, Couette flow problem with fluid injection, incorporating slip flow boundary conditions. In the interest of avoiding confusion, it is pointed out that the boundary condition at the upper (moving) plate is incorrect; in the notation of the reference, this boundary condition should be

$$u(y = L) = U - \xi_u(du/dy)_{y=L}$$

since the plate is on the positive y side of the fluid.

The solution of the problem then becomes

$$\frac{u}{U} = \frac{e^{R_e\eta} + R_e(\xi_u/L) - 1}{e^{R_e} - 1 + R_e(\xi_u/L)(e^{R_e} + 1)}$$

$$\frac{u_{s,0}}{U} = \frac{R_e(\xi_u/L)}{e^{R_e} - 1 + R_e(\xi_u/L)(e^{R_e} + 1)}$$

$$\frac{u_{s,L}}{U} = \frac{R_e(\xi_u/L)e^{R_e}}{e^{R_e} - 1 + R_e(\xi_u/L)(e^{R_e} + 1)}$$

Thus, as one intuitively would expect, the solutions exhibit no surprising singularities as $R_e(\xi_u/L) \to 1$.

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[1] Inman, R. M., "On porous-wall Couette flow under slip flow conditions," J. Aerospace Sci. 29, 1123 (1962).

Author's Reply to Comment by Donald M. Dix

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IN the foregoing note, Dix correctly points out the correct boundary conditions at the moving wall and gives the solution of the problem. Consequently, the author's assertion that velocities become indefinitely large as $R_e(\xi_u/L) \to 1$ is incorrect.

The correct solution also was kindly pointed out to the author earlier by S. H. Maslen of the Martin Marietta Company, and an errata was being prepared when the forementioned note came to the author's attention. Correct velocity profiles are shown in Fig. 1.

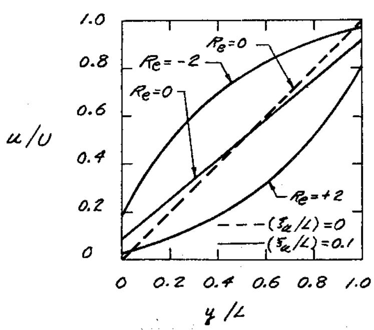

Fig. 1 Velocity distribution for porous-wall Couette flow in the slip flow regime

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Comments on the Validity of Ballistic Pendulum Measurements with Pulsed Plasma Accelerators

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A RECENT technical note[1] by Gooding et al. voiced dissatisfaction with the validity of momentum measurements obtained with ballistic pendulums constructed from several different materials. In the cited experiment, both electrically conducting and nonconducting materials were employed.

No points of contention with the reported data of the authors were found. However, it is felt that their general condemnation of the ballistic pendulum as a useful tool with pulsed plasma accelerators was somewhat arbitrary.

The Gooding experiment admittedly[2] was performed in "very rough and preliminary fashion" with no regard for the pendulum configuration. The results so obtained, as well as those of other investigators[3] who have used single-plane pendulum bobs, or variations thereof, are not at all surprising. In fact, it was the early recognition of the deficiencies of the conventional pendulum when applied to pulsed plasma acceleration which led to the unique approach to the problem described herein.

At Allison a ballistic pendulum has been developed, shown in Fig. 1, in which the entire discharge system of a rail-type plasma accelerator, including the electrodes, is surrounded completely by the pendulum bob. With this arrangement, any momentum components due to surface sputtering of the pendulum are cancelled through multiple collisions with the walls, with the possible exception of a negligible error introduced by impingement of sputtered mass onto the electrodes. The error necessarily would be somewhat larger in the case of the coaxial plasma gun due to the considerably larger capture area of the electrodes as contrasted with the small electrodes of the rail accelerator.

The deflection of the containing pendulum is determined by a technique that also is unique with Allison. An opaque shutter attached to the pendulum bob passes between a calibrated light source and a photocell. The amount of light reaching the photocell is inversely proportional to the pendulum deflection, and the cell output is recorded on a conventional data recorder. The actual pendulum deflection then is determined easily through reference to a calibration curve. More complete details of pendulum construction and data interpretation are given elsewhere.[4]

Several of the Gooding experiments have been reproduced, using identical materials in constructing the pendulums, but using the discharge-encompassing (closed) pendulum configuration. A basis for evaluating the results obtained with this configuration is provided by data obtained with the retrograde end of the cylindrical pendulum removed. The latter configuration provides an approximation to the conditions under which the Gooding data were obtained, although somewhat more capture and containment still should be experienced than with the simple single-face pendulum bob. The experiments were performed with a parallel rail accelerator coupled to a 6.4-μf capacitor, with the working media derived from electrically exploded 1-mil-diam silver wires.

No attempt was made to reproduce the data obtained with mica pendulums because of the wide variety of types of mica

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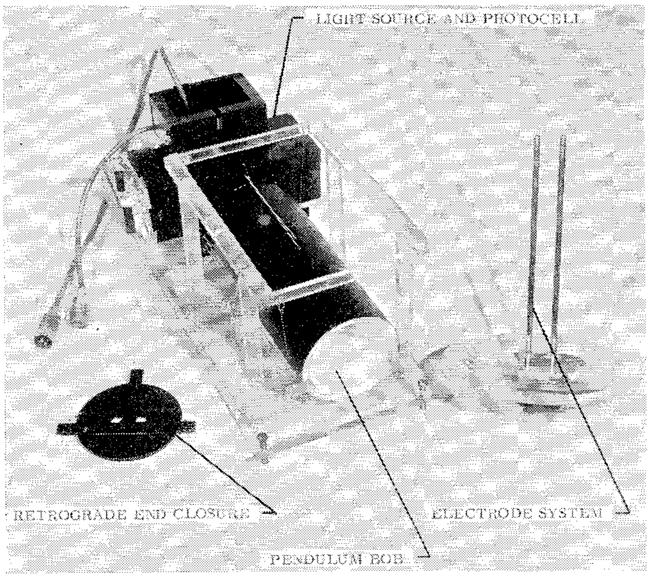

Fig. 1 Closed configuration ballistic pendulum components

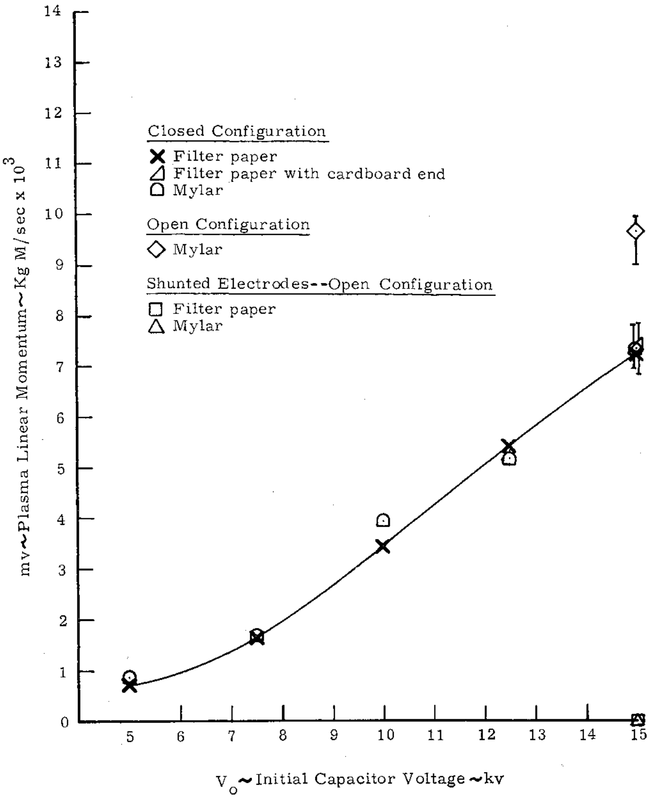

Fig. 2 Influence of ballistic pendulum material on momentum measurements with pulsed plasma accelerators; electrically nonconducting materials

which exist. In its place, data are presented for a mylar pendulum bob lined with chemically pure filter paper.

Possible pressure effects for all enclosed pendulums are eliminated by the inclusion of collimating vents that restrict any escaping particle velocities to directions normal to the axis of pendulum displacement. Image converter photographs indicate that this pressure problem is nonexistent.

Data for electrically nonconducting pendulum materials are presented graphically in Fig. 2, whereas the results obtained for electrically conducting pendulum materials are plotted to the same scale, for ease of comparison, in Fig. 3. In most cases, three recordings were taken at each point; the rms values are plotted, together with an indication of the spread observed in the several data points obtained at each

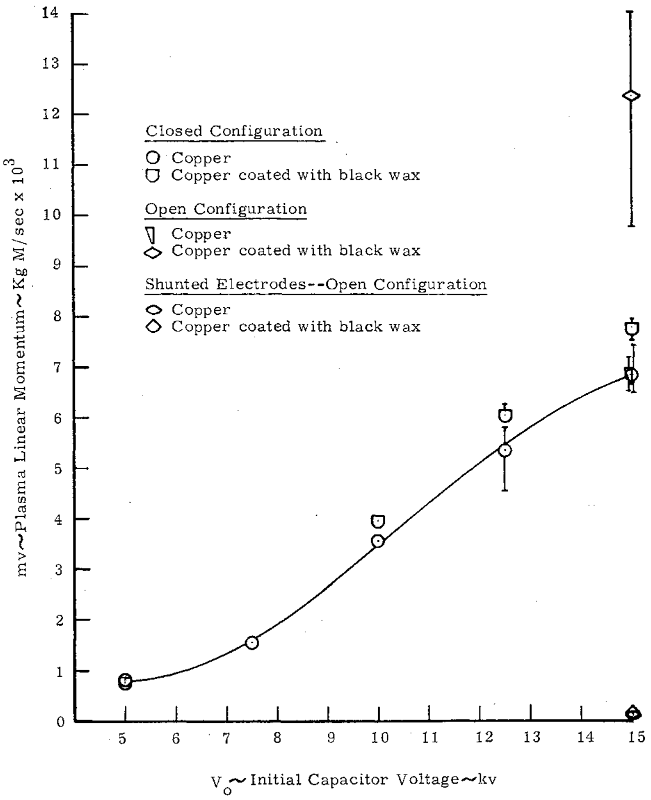

Fig. 3 Influence of ballistic pendulum material on momentum measurements with pulsed plasma accelerators; electrically conducting materials

setting. These experiments were conducted at a chamber pressure of 10^{-5} mm Hg. Previous calibrations showed that chamber pressure has little effect on momentum at pressures below this value. It is significant to note that the total data spread in most points with the closed pendulum was within the dimensions of the plotting symbol. The maximum variation of the rms values of the momenta recorded with the closed pendulum configuration was 15% for all materials tested. Data obtained with the open pendulum configuration were, however, completely unpredictable, in accordance with the observations of Lovberg.[2]

As an additional item of interest, shunted electrodes were used in conjunction with the open pendulum configuration to determine the interaction of the pendulum with the magnetic field associated with the discharge. A deflection was observed for the conducting materials which amounted to less than 3% of that obtained from exploded 1-mil-diam silver wires at the same initial stored energy. It is even more significant to note that no deflection occurred with pendulums constructed from electrically nonconducting materials.

It should be reiterated here that the authors share the skepticism of Gooding concerning data obtained from all open ballistic pendulum configurations. It is the conclusion of the present authors, however, that the closed pendulum configuration is indeed a useful tool when applied to pulsed plasma acceleration, and one that is capable of providing information obtainable by no other means.

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Reply by Authors to D. L. Clingman and T. L. Rosebrock

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IN a previous paper,[1] the authors wanted to illustrate that the simple ballistic pendulum is subject to error, and effects such as were observed should be considered before accepting impulse data obtained with pendulums. It was not intended to condemn ballistic pendulums per se; in fact, it was hoped to apply this technique in the authors' present work.

The solution to the ablation problem proposed by D. L. Clingman and T. L. Rosebrock seems to be a good one. Another approach simply is to reduce the energy density of the plasma at the surface of the pendulum by moving the pendulum away from the plasma accelerator so that the beam can spread radially and axially; however, the pendulum must become larger to collect all of the beam.

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Comment on "Invariant Two-Body Velocity Components"

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IN a previous paper,[1] Cronin and Schwartz discussed the invariant two-body velocity components of orbital body motion. Speculation was presented on possible useful applications of this property of invariance to the solution of two-body trajectory problems.

Since about three years ago, various studies of such applications have been conducted and are continuing. As noted in a XIIIth International Astronautical Congress paper,[2] additional literature on such applications is available.[3–14] Also, Newton's paper[15] is based upon this invariance property.

In general, the invariance of the specified orbital velocity components describes a circle that defines the orbital velocity vector in inertial space. This figure is known as the velocity hodograph, originally discovered by Hamilton and Möbius.

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Explicit Solution of the "Three-Moments Equation"

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THE solution of the "three-moments equation" given by the author[1] requires the search for eigenvalues and eigenvectors of a certain operator. To remove these difficulties and to find the explicit solution, one can substitute, instead of a finite-difference operator and boundary conditions, a new operator that transforms the boundary-value problem into an algebraic equation, without previously solving the problem. Commutation properties of this operator with Green's function make it possible to find the solution in classical form.

The "three-moments problem" has the notation

$$L_{mn}M_n = -G_m$$

$$M_0 = M_N = 0 \qquad n = 0,1,\ldots,N \tag{1}$$

where the finite-difference operator **L** and boundary conditions can be written as follows:

$$\left.\begin{aligned} L_{mn} &= \delta_{m+1,n} + 4\delta_{m,n} + \delta_{m-1,n} \\ M_0 &= M_N = 0 \end{aligned}\right\} \quad n = 0,1,\ldots,N \tag{2}$$

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* President.
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